INCENTIVES AND OPPORTUNITIES

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VERY PRELIMINARY

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Individual incentives may produce a costly misallocation of opportunities within an organization. Team incentives do not present this problem, however, they may require a higher cost for inducing the desired level of effort. In this paper we present a model in which a principal design a contract for n agents who may serve or pass a client to another agent. We show that the optimal contract depends on the size of the misallocation between agent and client. When this misallocation is above a certain threshold a team contract is more profitable than an individual contract. Moreover we find that, for a certain range of values, a team contract may perform better than an individual one despite of inducing lower effort. The reason is that in these cases the reduction in non-optimal referrals occurring under team incentives more than compensate the greater cost of effort. In the second part of the paper we study how the optimal contract should be designed when one of the n agents is a *superstar*. Our findings have implication for the optimal organizational form too.

Keywords: Team and Individual Incentives, Referrals, Gatekeepers.

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1. Introduction

Adam Smith in his famous (1776:15) example of a pin factory described the production of a pin in the following way:

> *"One man draws out the wire, another straights it, a third cuts it, a fourth points it, a fifth grinds it at the top for receiving the head: to make the head requires two or three distinct operations: to put it on is a particular business, to whiten the pins is another ... and the important business of making a pin is, in this manner, divided into about eighteen distinct operations, which in some manufactories are all performed by distinct hands, though in others the same man will sometime perform two or three of them."*

Smith used this example to explain what would become the central driver of economic progress for centuries to come: the division of labor and specialization. Previously, in a society where production was dominated by handcrafted goods, one man performed all the activities required during the production process. The result of labor division in Smith's example resulted in productivity increasing by 24000 percent, i.e. that the same number of workers made 240 times as many pins as they had been producing before the introduction of labor division.

This simple but at the same time revolutionary intuition of Adam Smith has represented a challenge for economic organizations. In fact, the dark side of the division of labor is the need for the employer to coordinate these specialized workers and to allocate tasks to the right person (and vice versa). Nowadays, with hyper-specialization, outsourcing and more flat organizations, this challenge has become even more complicated. Sometimes it is even difficult for the employer to know the competences and the resources that employees have and, therefore, to allocate the right person to the right place to do the right work. Misallocation of task-person (or more generally *opportunities*) happen everyday in many different organizations: public and private, big and small, manufacturer or service providers.

However, Adam Smith left us another important contribution that help employer: incentives can be used to align employees and employer interests and motivate employees to achieve those goals that employer desires. Thus, organizations thought a set of incentives are able to coordinate specialized workers. We can claim that one of the reasons why organizations exist is exactly to design adequate incentives. Or, using the words of Barnard (1938: 139):

> *"An essential element of organizations is the willingness of persons to contribute their individual efforts to the cooperative system. Inadequate incentives mean dissolution, or changes of organization purpose, or failure to cooperate. Hence, in all sorts of organizations the affording of adequate incentives becomes the most definitely emphasized task in their existence. It is probably in this aspect of executive work that failure is most pronounced."*

In case of misallocation of opportunities, the employer can design some incentive that rewards an agent that pass the task to the person that have the skill, competence or experience to take care of it. This agent will act as a *gatekeeper* in charge to detect who can take case of the opportunity. An opportunity can be everything: a task that needs to be done (e.g. in public administration), a service that has to be provided (e.g. in a law firm) or a product that has to be sold (e.g. in a bookstore). In all these examples, the opportunity will "knock at the door" of one employee that can *serve* or *pass* this opportunity to another employee. Which is the correct incentive to allocate efficiently opportunities within an organization? Do team incentives work better than individual ones? Under which conditions? These are the questions we aim to answer with this paper.

An individual incentive links wages to employees' performance. In this case there is no incentive for the employee to re-allocate the opportunity in case of a misallocation. Every employee will keep the opportunity for herself even if this is not optimal for the entire organization. On the contrary, a team incentive links wages to team performance. This incentive will lead to an optimal allocation (and eventually a re-allocation) of opportunities since each employee has the interest that opportunities are allocated optimally. However, as all the team incentive is subject to free-riding, there is a clear trade-off between these two incentives and for this reason it is important to find the condition under which is optimal for the employer to use one instead of another.

In this paper we present a principal-*n* agents model that serves as a stylized representation of the above interactions. The model includes sequential moves in three stages. In the first stage the principal defines an incentive-based contract that consists of a flat wage and a bonus. In the second stage, having observed the contract, the agents select the level of effort to exert in their job. In the third stage Nature selects the agents' position in the game and thus determine who has the opportunity to serve the client first (or execute the task). This agent can either serve the client or refer to another agent. If he serves the game ends; if he refers, then another agent has the opportunity to serve. For each agent the probability to succeed in serving the client depends on two components: an agent-specific component represented by the effort; and a position-specific component which capture the agent's productivity relative to the client characteristics. The value of such productivity is stochastic and it may differ across agents. The extent to which such productivity differs determine whether or not misallocation of opportunities exists.

We solve the model by backward induction and we compare the relative performance of two main type of contracts: an individual contract, in which the bonus is paid to the individual agent who is successful in serving the client; and a team contract, in which the bonus is paid to all agents in the case of success, independently of whom actually served the client. We find four main results: first, misallocation of opportunities combined with individual incentives my induce non-optimal referral; second, when misallocation of opportunities is not present, team incentives induce effort at a higher costs as compared to individual incentives; third, the optimal contracts depend on the size of misallocation of opportunities; and fourth, there exist a non-monotonic relationship between the agents' effort and the relative performance of the two contracts. In particular, with respect to the latter point, we find that there exist a whole range of values for the misallocation of opportunities for which a team contract is more profitable than an individual contract despite of inducing lower effort. The reason for this result is the following: for such a range of values the reduction in non-optimal referrals occurring under a team contract more than compensate the greater cost of effort.

In the second part of the paper we present an extension of the model by considering the case in which a superstar agent exists. We define two types of superstar: a position-specific superstar, i.e. an agent that in each position performs better in serving the client than any other agent moving in the same position; and a non-position-specific superstar, i.e. an agent who perform better in serving the client than any other agent independently of the position. We find that with a position-specific superstar the results of the model without superstar hold. Moreover we find that with a non-position-specific superstar the first-best contract turns out to be a gatekeeper contract in which all non-superstar agents are hired in order to serve the client to the superstar whenever they get the opportunity to. These findings have interesting implication for the emergence of hierarchies within the organization.

The paper contributes to the debate bringing new evidences to different streams of literature: i) principal-agent models with the trade off between individual and team incentives; ii) referrals under asymmetric information; iii) superstars and gatekeepers in economic organizations. In the economic literature several authors have discussed the design of incentives in principal-agent relationships (see Mirrlees (1975), Spence and Zeckhauser. (1971), Ross (1973) and Hölmstrom (1979)). This literature has emphasized the role of monetary compensations in aligning principal and agents' interests within organizations when information asymmetries exist. In these traditional models only one-task interaction are considered and the increased effort by the agent is assumed to be beneficial to the firm but costly to the agent. Principals face then the problem of how to design adequate incentives aimed at inducing greater effort by the agents under the assumption that effort is not verifiable. More recent contributions by Hölmstrom and Milgrom (1991) and Lal and Srinivasan (1993) have also modeled multitask principal-agent interactions where agents have to select effort along a vector of tasks. In all these models effort provision is the main and only object of incentive design. With respect to this literature the contribution of our paper is twofold. First, we develop multitask principal-n agent model in which incentives serve a double aims: (a) to induce effort provision and (b) to reduce opportunities misallocation. On this basis, we show that under reasonable assumptions a trade-off between these two aims exist. Second, we consider an economy in which information asymmetries exist both between the principal and the agents, and between the agents and the clients. This leads to a result that challenges the standard assumption (in principal-agent theory) on the monotonic relationship between profit and effort.

Hamilton et al., (2003) and Boning et al., (2007) provide empirical evidence suggesting that, in spite of the free-riding problems suggested by Alchian and Demsetz (1972), team incentives rise productivity, and that such an effect is greater, the greater the heterogeneity of the agents and the complexity of the production lines. In this respect our paper can provide an explanation of the results. Despite of the free-riding problem, in fact, team incentives may increase productivity via an increase in the efficiency of opportunities allocation.

Garicano (2000) model the transmission of tasks among agents at different layers of the hierarchy, with the quality of the agents as problem solvers being greater the more one goes up in the hierarchy. On this basis, it studies the trade-off between the cost and benefit of building new hierarchical layers. In our paper, on the contrary we take the number of layers as given and study the transmission of tasks among agents operating at the same layer.

Garicano and Santos (2004) model the same trade-off as our paper, i.e. between effort incentives and referral incentives. However, they do not study the problem of a principal who is to design incentives in the presence of such a trade-off. On the contrary, they study the relative performance of different ex ante referral agreement among agents.

Inderst and Ottaviani (2009) focus on misselling and its effect on the firm's reputation. They consider agents who perform two tasks: prospecting for customers and advising on the products suitability. Firms face a trade-off between the expected losses from misselling unsuitable products and the agency costs of providing marketing incentives. In their model, differently from our, they don't consider interactions and task transfers taking place among agents.

The remainder of the paper is organized as follows. Section 2 presents the model we will use for our analysis. Section 3 describes the trade off between individual and team incentives, and show under which conditions one contract is more profitable than the other. Section 4 extends our model to the case in which employees have different productivity and introduce a new contract called gatekeeper contract. Section 5 concludes. An attached appendix provide the proof of our results reported in Propositions 1 and 2.

2. The Model

We model the interaction between one client and a firm in which n (>1) agents work for a principal (P). The suffix i identifies the generic and *ex ante* identical agent. Figure 1 depicts the environment we have in mind. For instance, one can think about a department store managed by P in which n shop assistants work. Or, one can think about n retail outlets owned by P. Or even a real estate agency in which work n realtors under the supervision of P. In all these examples, each agent may have the *opportunity* to interact with the client: she can *serve* the client and supplies the requested service or *refer* the client to a second agent. If one (out of n) agent supplies successfully this service, then the client will pay x_S to the principal; if otherwise the agent fails, the client will pay $x_F (< x_S)$ to the principal.

[Figure 1 about here]

The timing of our model has three stages. In Stage 1, *P* specifies an incentive contract for the *n* agents. In Stage 2, having observed the incentive contract, each agent decides privately and simultaneously how much effort *e* to devote to her job. The cost of effort is $c(e)$ where $c'(e) > 0$ and $c''(e) > 0$. In Stage 3, Nature selects the first agent who has the opportunity to serve the client. Hereafter we call this agent as the *first mover* and we assume that each agent *i* has the same probability $p_i = \frac{1}{n}$ to be in this position. Then, the first mover can either serve the client –in this case the game ends- or refer the client to the most appropriate[1] agent among the remaining ones and thus leave her the opportunity to serve the client. We will refer to this second agent as the *second mover*.

The probability of success of each agent has two components. First, a *position-specific* component determined by Nature and modeled as a draw of two random variables $\lambda_1 \in [0,1]$ and $\lambda_2 \in [0,1]$ for the first and second mover respectively from a joint probability distribution $Z(\lambda_1, \lambda_2)$[2]; second, an *agent-specific* component determined by individual agents' effort. In more detail, the probabilities of success for the first and second mover are respectively $\lambda_1 f(e_1)$ and $\lambda_2 f(e_2)$, where $f(e_i)$ for $i = 1,2$ is a continuously differentiable function such that $f'(e_i) > 0$ and $f''(e_i) = 0$. The relative size of λ_1 and λ_2 depends on the information asymmetries that exist between the client and the agents. If information asymmetries are low, because for instance the characteristics of the item or of the service are well known or there is a brand that conveys information, the client will be likely to turn first to the best performing agent and thus $\lambda_1 \geq \lambda_2$. On the contrary, if information asymmetries are high, it may happen that $\lambda_1 \leq \lambda_2$ and *misallocation* of opportunity arises.

In Stage 1, P has to design an incentive contract for the agents. However, since both e_i and λ_i are not observable (or verifiable), *P* can only link this contract to the two observable outcomes:

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[1] MP:**Spiegare cosa intendiamo con most appropriate. Magari possiamo usare un altro termine che ora non mi viene in mente.**

[2] Notice that λ_1 and λ_2 depend on the type of service that is exchanged and on the level of competence/information required to the client for choosing the most appropriate agent. If, for instance, the service is a standard one, it is likely that client is able to identify the right agent and therefore $\lambda_1 \geq \lambda_2$. On the contrary, if there is an asymmetric information between client and agents, it may be that a misallocations occurs and $\lambda_1 \leq \lambda_2$.

agent's success or failure when she serves the client. Hence, we define a contract as a pair (w,b), where $w \geq 0$ is the baseline wage[3] and $b \geq 0$ is the bonus for the agent(s) in the case of success. In this paper we will focus on two contracts: an Individual Contract (w,b^{IC}) in which in case of success the bonus is paid only to the agent who served the client; and a Team Contract (w,b^{TC}) in which in case of success the bonus is paid to all agents. Table 1 summarizes the contingent payments to the agents and the principal under IC and TC.

[Table 1 about here]

Before going deeper into the analysis, we want to point out five assumptions embedded in our model. First, both the principal and the agents are risk neutral. Second, the *n* agents have already been hired, they cannot reject the contract offered[4]. Third, individual effort is observed by all agents (but not by P) and when referral takes place the service is passed to most appropriate agent among the others. Fourth, the second mover cannot refer to another agent but always serves. Fifth, both IC and TC provide non-negative wages and bonuses: it follows that *P* cannot use penalties. .

In the next two sections (2.1 and 2.2) we will solve our model by backward induction under IC and TC. We will study the first mover decision in Stage 3[5] and the effect of these two contracts on agents action decision. Then, we will analyze the effort choice problem faced by each agent in Stage 2: this will give us the agents' best response functions both under IC and TC. Finally, we will solve the optimal bonus choice problem faced by the principal in Stage 1.

2.1 Optimal Action, Effort and Bonus under Individual Contract

Under IC only the agent who serves the client can get the bonus. As a consequence while the first mover's income from referring in Stage 3 is equal to w with probability 1, the first mover's

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[3] Note that *w* goes to the agents even in case of failure in serving the client.
[4] This simplifies the analysis because it allows us to consider only an incentive compatibility constraint in the principal-agent problem, while neglecting the usual participation constraint. This simplification, however, does not affect our results.
[5] Given the above assumptions, second mover's decision is trivial, since it is always optimal to serve whenever she gets the opportunity to do so. Hence, in Stage 3, we will only study the decision problem faced by the first mover.

income from referring is equal to $w+b$ in the case of success and w in the case of failure. It follows that (proofs for all lemmas and propositions are reported in the Appendix):

LEMMA 1. *For any* $b \geq 0$, *serving is always optimal under IC*

Therefore, this contract may distort the matching between serving agent-client, i.e. the first mover will serve even if the second mover has an higher probability of success. This *misallocation* is costly for *P* and its size depends on the values of the random variables λ_1 and λ_2.

In Stage 2 each agent is selected as a first mover with the same probability $p = \frac{1}{n}$. Moreover, all agents are assumed to be ex-ante identical and they all face the exact same effort problem in Stage 2. Therefore, we can solve the latter by considering only a representative agent *i*.

Let e_i be the effort exerted by agent *i* and $\overline{\lambda}_1 = \int \lambda_1 dZ(\lambda_1, \lambda_2)$ be the expected value of the position-specific component of the productivity for the first mover. From LEMMA 1 it follows that *i*'s effort choice problem in Stage 2 is

$$\max_{e_i} \frac{1}{n}\{\overline{\lambda}_1 f(e_i)(w+b) + [1-\overline{\lambda}_1 f(e_i)]w\} + \left(\frac{n-1}{n}\right)w - c(e_i) \tag{1}$$

subject to $w \geq 0$, $b \geq 0$

where the first term in the objective function represents the expected income from being selected as first mover, the second term is the income in the case the agent is not selected as first mover, and the third term is the cost of effort. The solution to this maximization problem gives us the following first-order condition:

$$\frac{1}{n}\overline{\lambda}_1 f'(e_i)b = c'(e_i) \tag{2}$$

This equation shows that under IC agent *i* selects the level of effort in which the marginal benefit associated with an increase in the probability of earning the bonus is equal to the marginal cost of effort. Thus, the optimal effort depends on two factors: the bonus b and the expected quality of the opportunity $\overline{\lambda}_1$. The former is under the control of the principal, while the latter is a stochastic event. This optimality condition gives us *i*'s best response function, i.e. $e_i(b, \overline{\lambda}_1)$.

The problem of the principal in Stage 1 is

$$\max_b \overline{\lambda}_1 f(e_i)(x_S - b - nw) + [1 - \overline{\lambda}_1 f(e_i)](x_F - nw) \tag{3}$$

subject to $e_i = e_i(b, \overline{\lambda}_1)$, (4)

$w \geq 0$, $b \geq 0$, $x_S \geq x_F$, (5)

$x_S \geq b + nw$ and $x_F \geq nw$ (6)

where constraints in equations (6) ensure that *P* earns a non-negative profit. In the objective function the first term represents *P*'s expected net profit in the case of success and the second term represent *P*'s expected net profit in the case of failure. By substituting constraint (4) into the objective function, *P*'s problem can be rewritten as:

$$\max_b \overline{\lambda}_1 f(e_i(b, \overline{\lambda}_1))(x_S - b - nw) + [1 - \overline{\lambda}_1 f(e_i(b, \overline{\lambda}_1))](x_F - nw) \tag{7}$$

subject to constraints (5) and (6). Simple algebra shows that the optimal bonus b^{IC} is determined as the solution of the first order condition:

$$f'(e_i(b^{IC}, \overline{\lambda}_1))e_{ib}[x_S - x_F - b^{IC}] = f(e_i(b^{IC}, \overline{\lambda}_1))$$

where e_{ib} is the derivative of *i*'s best response function with respect to the bonus. By substituting b^{IC} back into (2) we obtain *i*'s optimal effort e_i^{IC} as a solution to:

$$\frac{1}{n}\overline{\lambda}_1 f'(e_i^{IC})b^{IC} = c'(e_i^{IC})$$

2.2 Optimal Action, Effort and Bonus under Team Contract

Under a TC all agents earn the bonus independently of the identity of who served the client. As a consequence the first mover's income is always equal to $w+b$ in the case of success and to w in the case of failure. The same is true for all the other agents. It follows that:

LEMMA 2. *In equilibrium under a TC, the first mover will serve if and only if* $\lambda_1 \geq \lambda_2$

The above condition suggests that under a TC referral takes place when the second mover has a higher probability of successfully serving than the first mover. Referral occurs when is optimal to do so and, therefore, there is no misallocation of opportunities. In other words, the distortion observed under IC is not present under a TC.

Similarly to the case of IC, in Stage 2 each agent has a probability of being selected as a first mover equal to $p = \frac{1}{n}$ and all agents face the exact same effort choice problem. As we previously did, we solve this problem for the representative agent *i*.

Let e_{-i} be the effort exercised by any agent other than *i* who is selected to play. LEMMA 2 implies that if $\lambda_1 \geq \lambda_2$ *i*'s expected income is:

$$\frac{1}{n}[\lambda_1 f(e_i)(w+b) + (1-\lambda_1 f(e_i))w] + \left(\frac{n-1}{n}\right)[\lambda_1 f(e_{-i})(w+b) + (1-\lambda_1 f(e_{-i}))w] \qquad (8)$$

where the first term represents the expected income when *i* is selected as the first mover and serves, while the second term is the expected income when *-i* is selected as first mover and serves. Similarly, if $\lambda_1 \leq \lambda_2$ *i*'s expected income is:

$$\frac{1}{n}[\lambda_2 f(e_{-i})(w+b) + (1-\lambda_2 f(e_{-i}))w] +$$
$$+\left(\frac{n-1}{n}\right)\left\{\lambda_2\left[\frac{1}{n-1}f(e_i) + \left(1-\frac{1}{n-1}\right)f(e_{-i})\right]b + w\right\} \qquad (9)$$

where we used the assumption according to which agents have an equal chance to be the first-best among the non-selected agents. In equation (9) the first term represents the expected income when *i* is selected as the first mover and refers, while the second term is the expected income when *-i* is selected as first mover and refers either to *i* or to another agent who is not *i*. Let $\underline{\lambda}_1 = \int_{\lambda_1 \geq \lambda_2} \lambda_1 dZ(\lambda_1, \lambda_2)$ and $\underline{\lambda}_2 = \int_{\lambda_1 \leq \lambda_2} \lambda_2 dZ(\lambda_1, \lambda_2)$ be the expected value of λ_1 and λ_2 conditional on $\lambda_1 \geq \lambda_2$ and $\lambda_1 \leq \lambda_2$ respectively. Then, by combining and rearranging the (8) and the (9), *i*'s effort choice problem in Stage 2 can be written as:

$$\max_{e_i} \frac{1}{n}[\underline{\lambda}_1 f(e_i) + \underline{\lambda}_2 f(e_{-i})]b +$$
$$+\left(\frac{n-1}{n}\right)\left\{\underline{\lambda}_1 f(e_{-i}) + \underline{\lambda}_2 \left[\frac{1}{n-1} f(e_i) + \left(1 - \frac{1}{n-1}\right) f(e_{-i})\right]\right\} b + w - c(e_i) \qquad (10)$$

subject to $\quad w \geq 0$, $b \geq 0$

The solution to this maximization problem gives us the following first-order condition:

$$\frac{1}{n}(\underline{\lambda}_1 + \underline{\lambda}_2) f'(e_i) b = c'(e_i) \qquad (11)$$

which is very similar to the one obtained under IC. Once again *i* selects the level of effort in which the marginal benefit associated with an increase in the probability of earning the bonus equals the marginal cost of effort. Again, the optimal effort level depends on two main components: the bonus b and the expected quality of the opportunity. However, under TC there is a positive probability that referral will take place; it follows that the expected value of the position-specific component of the productivity for both the first and the second mover (i.e. $\underline{\lambda}_1$ and $\underline{\lambda}_2$) affect the agent's effort choice. This condition of optimality gives us *i*'s best response function, $e_i(b, \underline{\lambda}_1, \underline{\lambda}_2)$.

The problem of the principal in Stage 1 is

$$\max_b \left\{ \frac{1}{n} [\underline{\lambda}_1 f(e_i) + \underline{\lambda}_2 f(e_{-i})] \right\} (x_S - nb - nw) +$$

$$+ \left(\frac{n-1}{n} \right) \left\{ \underline{\lambda}_1 f(e_{-i}) + \underline{\lambda}_2 \left[\frac{1}{n-1} f(e_i) + \left(1 - \frac{1}{n-1} \right) f(e_{-i}) \right] \right\} (x_S - nb - nw) +$$

$$+ \left\{ \frac{1}{n} [1 - \underline{\lambda}_1 f(e_i) - \underline{\lambda}_2 f(e_{-i})] \right\} (x_F - nw) +$$

$$+ \left(\frac{n-1}{n} \right) \left\{ 1 - \underline{\lambda}_1 f(e_{-i}) - \underline{\lambda}_2 \left[\frac{1}{n-1} f(e_i) + \left(1 - \frac{1}{n-1} \right) f(e_{-i}) \right] \right\} (x_F - nw) \qquad (12)$$

subject to $e_i = e_i(b, \underline{\lambda}_1, \underline{\lambda}_2)$, (13)

$e_{-i} = e_{-i}(b, \underline{\lambda}_1, \underline{\lambda}_2)$ (14)

$w \geq 0$, $b \geq 0$, $x_S \geq x_F$, (15)

$x_S \geq nb + nw$ and $x_F \geq nw$ (16)

where the first and the last two terms in the objective function represents *P*'s expected net profit in the case of success and failure respectively. Constraints (16) are once again included to ensure that the principal does not earn a negative profit. By substituting constraints (13) and (14) into the objective function, and given that under the assumption of agents' homogeneity $f(e_i) = f(e_{-i})$, *P*'s problem can be rewritten as:

$$\max_b (\underline{\lambda}_1 + \underline{\lambda}_2) f(e_i(b, \underline{\lambda}_1, \underline{\lambda}_2))(x_S - x_F - nb) + x_F - nw \qquad (17)$$

subject to constraints (15) and (16). Simple algebra shows that the optimal bonus b^{TC} is determined as the solution of the first order condition:

$$f'(e_i(b^{TC}, \underline{\lambda}_1, \underline{\lambda}_2)) e_{ib} (x_S - x_F - nb^{TC}) = nf(e_i(b^{TC}, \underline{\lambda}_1, \underline{\lambda}_2))$$

where e_{i_b} is the derivative of the agents' best response functions with respect to the bonus. By substituting b^{TC} back into (11) we obtain *i*'s optimal effort e_i^{TC} as a solution to:

$$\frac{1}{n}(\underline{\lambda}_1 + \underline{\lambda}_2) f'(e_i^{TC}) b^{TC} = c'(e_i^{TC})$$

3. Individual and Team Contract

In the previous section we saw that under TC the first mover always refers when the second mover has a higher probability of success. Therefore, the principal can easily align the agents' objectives with her own. This does not happen under IC, since the first mover has an incentive to always serve, independently on the probability of success of the second mover.

The relative profitability of these two contracts, however, does not depend only on the distortion caused by the agents' choice in the Stage 3. In particular, two additional factors need to be considered. First, the cost of the contracts defined as the sum of the bonuses and the wages paid to the agents. Second the agents' effort choice in Stage 2: under TC agents may have an incentive to exert less effort than under IC because of both the lower bonus that they earn in the case of success and the possibility of earning the bonus even without serving the client. In this sense there exists a trade-off between the selection of team-based incentives in order to favor referral when necessary, and the definition of sufficiently high bonuses at the individual level in order to induce effort. As the following examples illustrate, the condition on the preferability of one contract over the other does essentially depend on the size of expected misallocation, i.e. the expected productivity loss that the principal would incur by having the first mover serving the client with probability one. Formally, we define the latter as the ratio $\mu(\lambda) = (\underline{\lambda}_1 + \underline{\lambda}_2)/\overline{\lambda}_1$. Our measure captures the extent to which the productivity gain that P would get by letting the second mover to play is high relative to having only the first mover acting in Stage 3. The value of $\mu(\lambda)$ ranges from 1, when $\lambda_1 > \lambda_2$ (i.e. $\underline{\lambda}_1 = \overline{\lambda}_1$) and there is no misallocation, to ∞ when $\overline{\lambda}_1 = 0$ and misallocation is maximal.

Example 1: Suppose $f(e_i) = e_i$ and $c(e_i) = e_i^2/2$ for $i = A, B$, Z has a density function $z(\lambda)$ such that $\overline{\lambda}_1 = 2/5$, $\underline{\lambda}_1 = 2/5$ and $\underline{\lambda}_2 = 1/10$, and $n = 2$. This implies that the degree of misallocation is equal to $\mu(\lambda) = (\underline{\lambda}_1 + \underline{\lambda}_2)/\overline{\lambda}_1 = 5/4$. Under IC, given the parameters, we can solve the agents' first order condition reported in equation (2) to find $e_i = b/5$. Similarly, under TC, equation (11) implies $e_i = b/4$. Substituting these two best response functions into the principal's profit function and deriving the first order condition, we obtain that the optimal bonus under IC and TC is $b^{IC} = (x_S - x_F)/2$ and $b^{TC} = (x_S - x_F)/4$. Which in turn implies, according to the (2) and (11), that the optimal level of effort under IC and TC are:

$$e_i^{IC} = \frac{(x_S - x_F)}{10}, \qquad e_i^{TC} = \frac{(x_S - x_F)}{16}$$

for $i = A, B$. Thus the principal's final profit with IC and TC, call them π^{IC} and π^{TC} respectively, are:

$$\pi^{IC} = \frac{(x_S - x_F)^2}{50} + x_F - 2w, \qquad \pi^{TC} = \frac{(x_S - x_F)^2}{64} + x_F - 2w$$

This shows that given the specific form of $z(\lambda)$, IC is more profitable than TC. Moreover, the agents exert more effort under the former rather than the latter. Finally, the optimal bonus under IC is two times as big as the one under TC and does not depend on $z(\lambda)$, i.e. under TC the agents share in the optimal bonus that they would get individually under IC.

Example 2: Consider now the same setting as in Example 1 with the exception that *Z* has a density function $z(\lambda)$ such that $\overline{\lambda}_1 = 2/5$, $\underline{\lambda}_1 = 2/5$ and $\underline{\lambda}_2 = 3/5$. This implies that the degree of misallocation is greater than in Example 1 and equal to $\mu(\lambda) = (\underline{\lambda}_1 + \underline{\lambda}_2)/\overline{\lambda}_1 = 5/2$. In this case, under IC, nothing changes with respect to Example 1 so that the agents' best response function is $e_i = b/5$. Under TC, on the contrary, the higher value of $\underline{\lambda}_2$ implies $e_i = b/2$. Substituting these two best response functions into the principal's profit function and deriving the first order condition, we obtain that the optimal bonus under IC and TC is $b^{IC} = (x_S - x_F)/2$ and $b^{TC} = (x_S - x_F)/4$. Which in turn implies, according to the (2) and (11), that the optimal level of effort under IC and TC are:

$$e_i^{IC} = \frac{(x_S - x_F)}{10}, \qquad e_i^{TC} = \frac{(x_S - x_F)}{8}$$

for $i = A, B$. Thus, the principal's final profit with IC and TC are:

$$\pi^{IC} = \frac{(x_S - x_F)^2}{50} + x_F - 2w, \qquad \pi^{TC} = \frac{(x_S - x_F)^2}{16} + x_F - 2w$$

In this case, given the specific form of $z(\lambda)$, we find the opposite results with respect to Example 1, i.e. TC is more profitable than IC, and agents exert a more effort under TC than under IC. Interestingly, we find the optimal bonus for the two contracts is the same as in Example 1, i.e. the agents share in the bonus under TC. Hence, the result in terms of different profitability does not depend on the direct cost of the contract.

Example 3: Consider now the same setting as in Example 1 and 2 with the exception that Z has a density function $z(\lambda)$ such that $\overline{\lambda}_1 = 2/5$, $\underline{\lambda}_1 = 2/5$ and $\underline{\lambda}_2 = 4/15$. This implies that the degree of misallocation is equal to $\mu(\lambda) = (\underline{\lambda}_1 + \underline{\lambda}_2)/\overline{\lambda}_1 = 5/3$. In this case, under IC, nothing changes with respect to Examples 1 and 2. Under TC, on the contrary, the value of $\underline{\lambda}_2$ implies $e_i = b/3$ for $i = A, B$. Similarly to the previous cases the optimal bonus under IC and TC is $b^{IC} = (x_S - x_F)/2$ and $b^{TC} = (x_S - x_F)/4$. Which in turn implies, according to the (2) and (11), that the optimal level of effort under IC and TC are:

$$e_i^{IC} = \frac{(x_S - x_F)}{10}, \qquad e_i^{TC} = \frac{(x_S - x_F)}{12}$$

for $i = A, B$. Thus, the principal's final profit with IC and TC are:

$$\pi^{IC} = \frac{(x_S - x_F)^2}{50} + x_F - 2w, \qquad \pi^{TC} = \frac{(x_S - x_F)^2}{36} + x_F - 2w$$

In this case, differently from the two previous examples, we find that although agents exert less effort under TC than under IC, the former is more profitable than the latter. Moreover, similarly to Examples 1 and 2, we find that the two contracts cost the same. Combining these two results, we may conclude that, in this particular case, the greater profitability of TC is related to its greater capacity of efficiently allocating opportunities across agents. The value of $\underline{\lambda}_2$, in fact, implies that the gain that P would get by giving to the second mover the possibility to serve the client, more than compensate the cost due to the low level of effort.

[Figure 2 about here]

By considering Examples 1, 2 and 3 together, it is clear that the main factor driving the final results is the value of $\mu(\lambda)$. In particular, the greater $\mu(\lambda)$, the more profitable is TC relative to IC. Figure 2 offers a graphical representation of this result. In the graph we report the effort and profit under TC relative to the effort and profit under IC (i.e. $\underline{e} = e^{TC} / e^{IC}$ and $\underline{\pi} = \pi^{TC} / \pi^{IC}$) as a function of $\mu(\lambda)$.[6] The red line equal to 1 defines the set of points for which one of the two contracts performs relatively better than (or equal to) the other. For all the values above (below) the red line, in particular, both the effort and the profit under TC are greater (smaller) than the ones under IC. We also highlighted with three different colors the points associated with the previous examples.

As it is easy to see there exist two main thresholds for the value of $\mu(\lambda)$: first $\mu^* = \sqrt{2}$, beyond which the *profit* under TC is greater then the *profit* under IC; and second $\mu^e = 2$, beyond which the *effort* under TC is greater than the *effort* under IC. Interestingly, the fact that $\mu^* < \mu^e$ implies that there is a whole set of points (i.e. those for which $\mu^* \leq \mu \leq \mu^e$) for which TC generates a greater profit than IC in spite of inducing a lower effort. Example 3 is a combination of parameters that falls into this set. The intuition behind this result is the following: for this range of values the positive effect of TC in reducing the misallocation of opportunities more than compensate its cost in terms of effort provision. Obviously, this also implies that if the principal were to operate in this parameter space she would find it disadvantageous to consider effort

[6] For the easenees of representation the curve $\underline{\pi} = \mu(\lambda)^2 / 2$ has been drawn by assuming $x_F = 2w$. This, however, does not change the results.

inducement as the only target of incentive. The latter point, as we will argue below, has interesting policy implications.

The following proposition provides a generalization of the above result.

PROPOSITION 1. *Suppose* $f(e_i) = (\theta\, e_i^{\gamma})^{1/\gamma}$ *and* $c(e_i) = e_i^2 / 2$ *for* $i = 1,...,n$, *and* $z(\lambda) \geq 0$ *for all* $\lambda \in \Lambda$. *Then the following hold: (i)* $b^{TC} = b^{IC} / n$, *i.e. for any cumulative distribution of* λ_1 *and* λ_2, *i.e. the total cost of the contract is always the same; (ii) there exist a* $\mu^e = n$ *such that if* $\mu(\lambda) \leq \mu^e$ *then* $e_i^{IC} \geq e_i^{TC}$ *for* $i = 1,...,n$; *and (iii) there exist a* $\mu^* = \sqrt{n}$ *such that if* $\mu(\lambda) \leq \mu^*$ *then* $\pi^{IC} \geq \pi^{TC}$.

We believe that the most interesting result that is contained in Proposition 1 is the non-monotonic relation between the principal's profit and the agents' effort. In most of the literature on economic incentives, in fact, such monotonic relation has been often assumed and the focus has been the role of monetary compensations in solving effort-based information asymmetries. This, at least theoretically, has led individual-based incentives to be largely preferred over team-based ones. In our model, however, effort turns out to be only one of the factors affecting the design of monetary compensations, the avoidance of opportunity misallocation being the other. As a result we find that team-based incentives, which perform relatively better than individual-base ones in reducing misallocation, are preferable for a wider range of parameter than if misallocation did not play a role. This outcome is relevant for two reasons: first, it can explain why the standard model based on profit as a monotonic function of effort generally underestimates the amount of team-based incentives observed in the real world; and second it can provide managers with an extended set of variables that needs to be taken into account when designing incentives at the firm level.

4. Extension: Superstars

In the previous sections we assumed the stochastic component in the probability of success to be both position-specific and unrelated to the agent's identity, *i.e.* its value only depended on whether an agent was selected to play as first or second mover. This essentially implied that before Nature had made her move the game was perfectly symmetric. The expected values of λ_1

and λ_2 were essentially determined by the type of service being offered in conjunction with the characteristics of the client. This implied that the same two agents could perform better or worse relative to each other depending on the information asymmetries existing between them and the client. In this section we are going to remove this assumption.

In most organizations, in fact, it is reasonable to expect that some agents are always better than others in serving the clients. This could be the case of a civil servant who is much more knowledgeable in terms of bureaucratic practices than his colleagues, or a shop assistant who is particularly skilful in understanding the preferences of her client. In all these cases the profit of the employer would significantly increase if he were able to let these *superstar* agents (rather than their peers) deal with the opportunities as they appears. This section explicitly look at the contract that is most effective in obtaining this result.

In our setting the superstar can be introduced in two ways. First, we can consider a *position-specific superstar*, i.e. an agent that in each position (i.e. as first or second mover) performs better than any other agent in the team moving in the same position. Let λ_{1ss} and λ_{2ss} be the superstar's stochastic component in the probability of success when moving as first and second mover respectively. Analogously, let λ_{1i} and λ_{2i} be the stochastic component of any other agent *i* who is a member of the team and is not a superstar. On this basis, a *position-specific superstar* exists if, for any $i \neq ss$, $\lambda_{1ss} > \lambda_{1i}$ and $\lambda_{2ss} > \lambda_{2i}$. In this case, it is easily shown that the results reported in Proposition 1 holds. When a position-specific superstar exists, in fact, there still is the possibility that some misallocation of opportunity arises, and depending on the expected size of such misallocation the effect of TC in reducing mismatching will make the latter more or less profitable relative to IC.

In order to illustrate the last point let us suppose that, for a given shape of $Z(\lambda_1, \lambda_2)$, the following sequence of inequalities holds: $\lambda_{2ss} > \lambda_{2i} > \lambda_{1ss} > \lambda_{1i}$. Under this assumption, although *ss* does better than *i* in both positions, it is clear that *i* does better than *ss* when the nature selects the latter to play as first mover. If this combination reveals and IC is offered to the agents, *ss* will find it optimal to serve under Lemma 1 and misallocation of opportunity will exist. Such misallocation, which is costly to the principal, could be reduced if TC were offered instead. At the same time, however, TC would induce lower efforts in Stage 2, in because of the smaller bonus that is paid out. Overall, the net result of such a trade-off, will essentially depend on the degree of misallocation, with TC becoming relatively more profitable than IC the more severe the risk of misallocation.

The result is different, however, if we consider the case of a non-position-specific *superstar*, i.e. an agent who perform better than any other agent in the team independently of the position. Formally, this condition can be introduced by letting the stochastic component in the probability of success to be agent-specific, i.e. $\lambda_i \in [0,1]$ for any agent i in the team. On this basis, a *superstar* exists if, for any $i \neq ss$, $\lambda_{ss} > \lambda_i$. Under this assumption it can be shown that TC perform always better than IC. Moreover, it can be proved that a third type of contract in which all the non-superstar agents operate as gatekeeper – i.e. they always refer to the superstar in exchange for a fixed payment, is the first best.

Let's consider first the relative performance of IC and TC. Under IC, similarly to the case without superstar, only the agent who serves the client earns the bonus. Hence, Lemma 1 holds and for any $b \geq 0$ serving is always optimal in Stage 3. This implies that with the presence of a non-position-specific superstar, and under the assumption that all agents are equally likely to be selected as first mover, IC is highly likely to be affected by misallocation. As long as information asymmetries exist between the client and the agents, in fact, there is (n-1)/n probability that the agents actually serving the client will not be the superstar. Moreover, the fact that *ss* and *i* are identical with the only exception of $\lambda_{ss} > \lambda_i$ implies that, for any $b \geq 0$, *ss* has a greater marginal benefit of effort than *i*, and will thus select a higher level of effort in Stage 2. This will exacerbate further the distorting effect generated by IC.

Under TC, on the contrary, all agents earn the exact same bonus in the case of success. As a consequence referral will occur so as to maximize the probability of success of the whole team. This implies that, if a non-position-specific superstar exists, all non-superstar agents will find it optimal to refer in Stage 3 and *ss* will always be the one serving the client in equilibrium. Knowing that, all non-superstar agents will select zero effort in Stage 2, whereas *ss* will select a strictly positive effort. In this case, obviously, TC will still be characterized by an additional cost relative to IC, which is due the n-times smaller bonus that is paid in the case of success. However, such a cost will be compensated by the fact that under TC *ss* serves the client with a probability that is n times greater than the one under IC, therefore making the marginal benefit of effort (and thus the optimal effort) equal in the two cases. As a result TC will enable the principal to minimize distortions without compromising the effort choices in Stage 2, and will thus generate a profit which is strictly greater than the one under IC.

Although in the presence of a non-position-specific superstar TC performs always better than IC, it is important to notice that this result comes at some cost from an efficiency point of view. In particular, since under TC the team bonus is the same for all agents independently of their productivity, there exist n-1 agents that earn a positive rent in equilibrium. All non-superstar agents, in fact, select zero effort in Stage 2 and always refer to *ss* in Stage 3, with the consequent positive probability of earning a bonus at no cost. The cause of such inefficiency is obviously the information asymmetries that exist between the principal and the agents, which prevent the former to contract bonuses conditional on the productivity of the latter. If such information asymmetries were not there, a first best contract with no equilibrium rents could be offered.

In order to characterize the first best contract let us assume that principal P knows who the superstar is. Moreover, for the sake of simplicity, let us as assume that all the non-superstar agents in the group are identical. On this basis, P could offer the following two contracts: to the superstar he offers a fixed wage plus an individual bonus $b \geq 0$ to serve whenever she gets the opportunity to; to the non-superstar agents he instead offers a fixed wage to refer to the superstar whenever they get the opportunity to. In this context, thus, the non-superstar agents act merely as gatekeepers whose main aim is to point the client to the best performing agent (i.e. the superstar). This contractual arrangement is what we call a gatekeeper contract (GC)

The solution of the model under GC is straightforward. In Stage 3, similarly to TC, it is always the superstar who serves the client. The main difference is that, while under TC this result was the outcome of the incentive scheme, in this case it derives from a direct prescription included in the contract. For the sake of simplicity we are not modeling the process of contract enforcement for non-superstar agents. We are simply assuming that, since referral is cost-free in our model, non-superstar agents have no incentive not to refer.

In Stage 2 *ss* solves the same problem as the one under TC. Also in this case *ss* will serve the client with probability one, and this will obviously increase the marginal product of her effort relative to IC. In addition to this, since under GC the bonus is individual, *ss*'s effort will be less costly to be induced on the side of the principal as compared to TC, because all the marginal benefit of effort will be actually earned by the agent performing the action. The combination of these two effects will enable the principal to extract more effort from ss than under IC and TC, and as a consequence to earn a greater profit. At the same time, no equilibrium rents will be earned by the non-superstar agents who will only act as gatekeepers for *ss*. In this sense, when a non-position-specific superstar is introduced, GC represents the first best contract.

The following proposition provide a general result for the performance of GC relative to both IC and TC with non-position-specific superstar:

PROPOSITION 2. *Suppose* $f(e_i) = (\theta\ e_i^{\gamma})^{1/\gamma}$ *and* $c(e_i) = e_i^2/2$ *for any* i *who is not the superstar. Moreover, suppose that a non-position-specific superstar ss exists, and that* $f(e_{ss}) = (\theta\ e_{ss}^{\gamma})^{1/\gamma}$ *and* $c(e_{ss}) = e_{ss}^2/2$. *Then, for any value of* λ_i *and* λ_{ss} *such that* $\lambda_{ss} > \lambda_i$, *the following hold: (i)* $b^{GC} = b^{IC} = nb^{TC}$, *i.e. under GC P offers the same bonus as under IC ; (ii)* $e_{ss}^{GC} > e_{ss}^{TC} = e_{ss}^{IC}$, *i.e. the superstar selects the highest effort under GC; and (iii)* $\pi^{GC} > \pi^{TC} > \pi^{IC}$, *i.e. GC is the first best contract.*

The results of Proposition 2 have interesting implications for the form of the organization. What we call a GC, in fact, is *tantamount* to the introduction of some form of hierarchy in the organization, whereas in particular two main layers can be considered: the first one, downstream, consisting of the non-superstar agents who constantly refer to the superstar whenever they get the opportunity to serve the client; and the second one, upstream, consisting of the superstar who actually serves the client. From this perspective, the main implications of our model can be summarized as follows: first, when the superstar is non-position-specific and information asymmetries between the principal and the agents are low, a hierarchy represents the first best solution for the principal; and second, when the superstar is position-specific and/or some information asymmetries between the principal and the agents exist, a hierarchy is not viable and a flat organization relying on incentive-based contracts is preferable[7]. In this sense, it is interesting to notice that the necessary condition for a hierarchy to be convenient is very restrictive. On this basis the prediction of our model is that in the businesses to which our model applies (e.g. realtors, department stores, chain shop) we should observe much less hierarchies than flat organizations.

[7] **This result is in line with Aghion and Tirole (1997): spiegare brevementet**.

5. Discussion

Modern organizations are characterized by hyper-specialization and massive division of labor. In such contexts misallocation of task-person or more generally of opportunities happens frequently. One of the main instruments that managers have in order to reduce the costs of misallocation is incentives.

An individual incentive links wages to employees' performance. In this case there is no incentive for the employee to re-allocate the opportunity in case of a misallocation. Every employee will keep the opportunity for herself even if this is not optimal for the entire organization. On the contrary, a team incentive links wages to team performance. This incentive will lead to an optimal allocation (and eventually a re-allocation) of opportunities since each employee has the interest that opportunities are allocated optimally. At the same time, however, team incentives are subject to free-riding therefore making individual effort more expensive to be induced. Overall, when misallocation of opportunities exist, there is a clear trade-off between individual and team incentives.

This paper present a principal-n agents model designed to analyze this trade-off and show under which conditions one contract is preferable to the other. We characterize the optimal contract varying the size of the misallocation of opportunity and we present some implications of our finding. In addition, we study how the optimal contract should be designed when one of the n agent is a superstar. This latter finding has implications for the organizational form too.

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Appendix A.

PROOF OF LEMMA 1. Let $m_{1,s}$ and $m_{1,r}$ be the first mover's income from serving and referring at Stage 3 respectively. Under IBIS $m_{1,s}$ and $m_{1,r}$ can be written as:

$$m_{1,s} = \lambda_1 f(e_1)(w+b) + [1-\lambda_1 f(e_1)]w$$

$$m_{1,r} = w$$

where $m_{1,s} \geq m_{1,r}$ for any $b \geq 0$. □

PROOF OF LEMMA 2. Under TC $m_{1,s}$ and $m_{1,r}$ can be written as:

$$m_{1,s} = \lambda_1 f(e_1)(w+b) + [1-\lambda_1 f(e_1)]w$$

$$m_{1,r} = \lambda_2 f(e_2)(w+b) + [1-\lambda_2 f(e_2)]w$$

where $m_{1,s} \geq m_{1,r}$ if and only if $\lambda_1 f(e_1) \geq \lambda_2 f(e_2)$. Since the two agents are assumed to be identical they will exert the same level of effort in equilibrium, i.e. $f(e_1^{TC}) = f(e_2^{TC})$ where e_j^{TC} for $j = 1,2$ is agent *j*'s optimal level of effort under TC. Thus, the previous condition reduces to $\lambda_1 \geq \lambda_2$.□

PROOF OF PROPOSITION 1. First we solve the model under IC. From (2), *i*'s best response function is:

$$e_i = \frac{1}{n}\overline{\lambda}_1 \theta^{1/\gamma} b \qquad \text{(A1)}$$

Given (A1), P's maximization problem in the (7) can be rewritten as:

$$\max_b \overline{\lambda}_1 \left[\theta \left(\frac{1}{n}\overline{\lambda}_1 \theta^{1/\gamma} b\right)^{\gamma}\right]^{1/\gamma} (x_S - x_F - b) + x_F - nw \qquad \text{(A2)}$$

subject to constraints (5) and (6). The first-order condition for problem (A2) gives us the optimal bonus under IC:

$$b^{IC} = \frac{x_S - x_F}{2} \qquad \text{(A3)}$$

By substituting (A3) into (A1), the optimal effort under IC is:

$$e_i^{IC} = \frac{\overline{\lambda}_1 \theta^{1/\gamma}(x_S - x_F)}{2n} \qquad \text{(A4)}$$

Finally, by substituting (A3) into the objective function of (A2) we get the profit under IC:

$$\pi^{IC} = \frac{\overline{\lambda}_1^2 \theta^{2/\gamma} (x_S - x_F)^2}{4n} + x_F - nw \tag{A5}$$

Consider now a TC. From (11), *i*'s best response function is:

$$e_i = \frac{1}{n} (\underline{\lambda}_1 + \underline{\lambda}_2) \theta^{1/\gamma} b \tag{A6}$$

Given (A6), P's maximization problem in the (17) can be rewritten as:

$$\max_b (\underline{\lambda}_1 + \underline{\lambda}_2) \left\{ \theta \left[\frac{1}{n} (\underline{\lambda}_1 + \underline{\lambda}_2) \theta^{1/\gamma} b \right]^{\gamma} \right\}^{1/\gamma} [x_S - x_F - nb] + x_F - nw \tag{A7}$$

subject to constraints (15) and (16). The first-order condition for problem (A7) gives us the optimal bonus under TC:

$$b^{TC} = \frac{x_S - x_F}{2n} \tag{A8}$$

By substituting (A8) into (A6), the optimal effort under TC is:

$$e_i^{TC} = \frac{(\underline{\lambda}_1 + \underline{\lambda}_2) \theta^{1/\gamma} (x_S - x_F)}{2n^2} \tag{A9}$$

Finally, by substituting (A8) into the objective function of (A7) we get the profit under TC:

$$\pi^{TC} = \frac{(\underline{\lambda}_1 + \underline{\lambda}_2)^2 \theta^{2/\gamma} (x_S - x_F)^2}{4n^2} + x_F - nw \tag{A10}$$

Point *(i)* follows from (A3) and (A8). By comparing (A4) and (A9) it is easy to see that $e_i^{IC} \geq e_i^{TC}$ if and only if $\mu(\lambda) = (\underline{\lambda}_1 + \underline{\lambda}_2) / \overline{\lambda}_1 \leq n = \mu^e$. This proofs point *(ii)*. Similarly, by comparing (A5) and (A10) it is easy to see that $\pi^{IC} \geq \pi^{TC}$ if and only if $\mu(\lambda) = (\underline{\lambda}_1 + \underline{\lambda}_2) / \overline{\lambda}_1 \leq \sqrt{n} = \mu^*$. This proofs point *(iii)*. □

PROOF OF PROPOSITION 2. First we solve the model under IC. When IC is offered Lemma 1 holds and for any $b \geq 0$ serving is always optimal in Stage 3. This implies that in Stage 2 the superstar *ss* and any other generic agent *i* solve the following problems:

$$\max_{e_{ss}} \frac{1}{n} \{ \lambda_{ss} f(e_{ss})(w + b) + [1 - \lambda_{ss} f(e_{ss})] w \} + \left(\frac{n-1}{n} \right) w - c(e_{ss})$$

$$\max_{e_i} \frac{1}{n}\{\lambda_i f(e_i)(w+b)+[1-\lambda_i f(e_i)]w\}+\left(\frac{n-1}{n}\right)w-c(e_i)$$

subject to $w \geq 0$, $b \geq 0$

These problems are very similar to the one reported in the model without superstars and give us the following first-order conditions:

$$\frac{1}{n}\lambda_{ss} f'(e_{ss})b = c'(e_{ss}) \tag{A11}$$

$$\frac{1}{n}\lambda_i f'(e_i)b = c'(e_i) \tag{A12}$$

By substituting into the (A11) and (A12) the corresponding explicit functions we obtain the following best response functions:

$$e_{ss} = \frac{1}{n}\lambda_{ss}\theta^{1/\gamma}b \tag{A13}$$

$$e_i = \frac{1}{n}\lambda_i\theta^{1/\gamma}b \tag{A14}$$

P's problem in Stage 1 under IC can thus be written as follows:

$$\max_b \frac{n-1}{n}\{\lambda_i f(e_i)(x_S-b-nw)+[1-\lambda_i f(e_i)](x_F-nw)\}+ \tag{A15}$$

$$+\frac{1}{n}\{\lambda_{ss} f(e_{ss})(x_S-b-nw)+[1-\lambda_{ss} f(e_{ss})](x_F-nw)\}$$

subject to $e_i = \frac{1}{n}\lambda_i\theta^{1/\gamma}b$, $e_{ss} = \frac{1}{n}\lambda_{ss}\theta^{1/\gamma}b$, $\lambda_{ss} > \lambda_i$ (A16)

$w \geq 0$, $b \geq 0$, $x_S \geq x_F$, (A17)

$x_S \geq b+nw$ and $x_F \geq nw$ (A18)

By substituting constraints (A16) into the objective function, and taking into account that $f(e_i) = (\theta\ e_i^{\gamma})^{1/\gamma}$ and $f(e_{ss}) = (\theta\ e_{ss}^{\gamma})^{1/\gamma}$, the previous problem can be rewritten as follows:

$$\max_b \frac{n-1}{n}\left\{\lambda_i\left[\theta\left(\frac{1}{n}\lambda_i\theta^{1/\gamma}b\right)^{\gamma}\right]^{1/\gamma}(x_S-x_F-b)+x_F-nw\right\}+ \tag{A19}$$

$$+\frac{1}{n}\left\{\lambda_s\left[\theta\left(\frac{1}{n}\lambda_s\theta^{1/\gamma}b\right)^{\gamma}\right]^{1/\gamma}(x_S-x_F-b)+x_F-nw\right\}$$

subject to constraints (A17) and (A18). The first-order condition for problem (A19) gives us the optimal bonus under IC:

$$b^{IC} = \frac{x_S - x_F}{2} \tag{A20}$$

Notice that b^{IC} is the same as in the model without superstar. By substituting (A20) into (A13) and (A14), *ss* and *i*'s optimal efforts under IC are:

$$e_s^{IC} = \frac{\lambda_{ss}\theta^{1/\gamma}(x_S - x_F)}{2n} \tag{A21}$$

$$e_i^{IC} = \frac{\lambda_i\theta^{1/\gamma}(x_S - x_F)}{2n} \tag{A22}$$

Finally, by substituting (A20) into the objective function of (A19) we get the profit under IC:

$$\pi^{IC} = \left[\frac{(n-1)\lambda_i^2\theta^{2/\gamma} + \lambda_{ss}^2\theta^{2/\gamma}}{n}\right]\frac{(x_S - x_F)^2}{4n} + x_F - nw \tag{A23}$$

Let's consider now TC. Under TC, as previously shown, referral will occur so as to maximize the probability of success of the whole team. It follows that if a non-position specific superstar exists, in equilibrium under TC it is always the superstar who serves the client. In order to see why let us consider that, under TC with superstar, $m_{i,s}$ and $m_{i,r}$ for any $i \neq ss$ can be written as:

$$m_{i,s} = \lambda_i f(e_i)(w+b) + [1 - \lambda_i f(e_i)]w \tag{A24}$$

$$m_{i,r} = \lambda_{ss} f(e_{ss})(w+b) + [1 - \lambda_{ss} f(e_{ss})]w \tag{A25}$$

where (A25) derives from $\lambda_{ss} > \lambda_i$ for any $i \neq ss$, *i.e.* whenever *ss* is not selected as the first mover she is always the first-best among the non-selected agents. Similarly, $m_{ss,s}$ and $m_{ss,r}$ can be written as:

$$m_{ss,s} = \lambda_{ss} f(e_{ss})(w+b) + [1 - \lambda_{ss} f(e_{ss})]w \tag{A26}$$

$$m_{ss,r} = \lambda_i f(e_i)(w+b) + [1 - \lambda_i f(e_i)]w \tag{A27}$$

By comparing equations (A24) and (A25) we find that $m_{i,s} \geq m_{i,r}$ if and only if

$$\lambda_i f(e_i) \geq \lambda_{ss} f(e_{ss}) \tag{A28}$$

Since $\lambda_{ss} > \lambda_i$ by assumption, condition (A28) would hold only if $f(e_i) > f(e_{ss})$ in equilibrium. Let's assume that this is true, and in particular that

$$f(e_i^{TC}) > f(e_{ss}^{TC}) \tag{A29}$$

Given $f'(.) > 0$, condition (A29) necessarily implies $e_i^{TC} > e_{ss}^{TC}$. Since both *ss* and *i* optimize, they will select their level of effort at the point in which the marginal benefit equals the marginal cost. We know by assumption that both agents face the same effort cost function. Hence, for condition (A29) to hold, it must necessarily be the case that *i*'s marginal benefit of effort is greater than *ss*'s one. Since the agent's own effort plays a role only when they serve we can derive *i*'s and *ss*'s marginal benefit of effort by deriving the (A24) and (A26) over e_i and e_{ss} respectively. On this basis we obtain that condition (A29) is satisfied if and only if:

$$\lambda_i f'(e_i)b > \lambda_{ss} f'(e_{ss})b \tag{A30}$$

where since $f'(e_i) = f'(e_{ss})$ by assumption, the (A30) reduces to $\lambda_i > \lambda_{ss}$. But this is impossible. Hence it must necessarily be that $f(e_i^{TC}) < f(e_{ss}^{TC})$, which in turn implies that in equilibrium *i* finds it always optimal to refer. By applying the same reasoning to *ss* it is trivial to show that the latter finds it always optimal to serve.

From this result it follows that in Stage 2 *ss* and *i* solve the following two problems

$$\max_{e_{ss}} \lambda_{ss} f(e_{ss})(w+b) + [1-\lambda_{ss} f(e_{ss})]w - c(e_{ss}) \tag{A31}$$

$$\max_{e_i} \lambda_{ss} f(e_{ss})(w+b) + [1-\lambda_{ss} f(e_{ss})]w - c(e_i)$$

subject to $w \geq 0$, $b \geq 0$

which give as first-order conditions:

$$\lambda_{ss} f'(e_{ss})b = c'(e_{ss}) \tag{A32}$$

$$-c'(e_i) = 0 \tag{A33}$$

By substituting into the (A32) and (A33) the corresponding explicit functions we obtain the following best response functions:

$$e_{ss} = \lambda_{ss}\theta^{1/\gamma} b \tag{A34}$$

$$e_i = 0 \tag{A35}$$

P's problem in Stage 1 under TC will thus take the following form:

$$\max_{b} \lambda_{ss} f(e_{ss})(x_S - nb - nw) + [1-\lambda_{ss} f(e_{ss})](x_F - nw) \tag{A36}$$

subject to

$$e_{ss} = \lambda_{ss}\theta^{1/\gamma} b \tag{A37}$$

$$w \geq 0,\ b \geq 0,\ x_S \geq x_F, \tag{A38}$$

$$x_S \geq b + nw \text{ and } x_F \geq nw \quad \text{(A39)}$$

By substituting constraints (A37) into the objective function, and taking into account that $f(e_{ss}) = (\theta\, e_{ss}^{\gamma})^{1/\gamma}$, the previous problem can be rewritten as follows

$$\max_b \lambda_{ss}\left[\theta\,\left(\lambda_{ss}\theta^{1/\gamma}b\right)^{\gamma}\right]^{1/\gamma}\left(x_S - x_F - nb\right) + x_F - nw \quad \text{(A40)}$$

subject to constraints (A38) and (A39). The first-order condition for problem (A40) gives us the optimal bonus under TC:

$$b^{TC} = \frac{x_S - x_F}{2n} \quad \text{(A41)}$$

which is once again equal to b^{TC} in the model without superstar. By substituting (A41) into (A34), *ss*'s optimal effort under TC is:

$$e_{ss}^{TC} = \frac{\lambda_{ss}\theta^{1/\gamma}(x_S - x_F)}{2n} \quad \text{(A42)}$$

By substituting (A41) into the objective function of (A40) we then get the profit under TC:

$$\pi^{TC} = \frac{\lambda_{ss}^{2}\theta^{2/\gamma}(x_S - x_F)^2}{4n} + x_F - nw \quad \text{(A43)}$$

Finally, let us consider a GC. In Stage 3, by contract, it is always the superstar who serves the client. It follows the non-superstar agents do not select any effort in Stage 2. ss, on the contrary, will solve the same problem as under TC, which obviously gives as first-order condition equation (A34). In Stage 1 P's problem will thus becomes the following:

$$\max_b \lambda_{ss} f(e_{ss})(x_S - b - nw) + [1 - \lambda_{ss} f(e_{ss})](x_F - nw) \quad \text{(A44)}$$

subject to $e_{ss} = \lambda_{ss}\theta^{1/\gamma}b$ (A45)

$$w \geq 0,\ b \geq 0,\ x_S \geq x_F, \quad \text{(A46)}$$

$$x_S \geq b + nw \text{ and } x_F \geq nw \quad \text{(A47)}$$

where the main difference with TC is that in this case only one bonus is paid. By substituting constraints (A45) into the objective function, and taking into account that $f(e_{ss}) = (\theta\, e_{ss}^{\gamma})^{1/\gamma}$, the previous problem can be rewritten as follows:

$$\max_b \lambda_{ss}\left[\theta\,\left(\lambda_{ss}\theta^{1/\gamma}b\right)^{\gamma}\right]^{1/\gamma}\left(x_S - x_F - b\right) + x_F - nw \quad \text{(A48)}$$

subject to constraints (A46) and (A47). The first-order condition for problem (A48) gives us the optimal bonus under GC:

$$b^{GC} = \frac{x_S - x_F}{2} \tag{A49}$$

By substituting (A49) into (A34), *ss*'s optimal effort under GC is:

$$e_{ss}^{GC} = \frac{\lambda_{ss}\theta^{1/\gamma}(x_S - x_F)}{2} \tag{A50}$$

Finally, by substituting (A49) into the objective function of (A48) we get the profit under GC:

$$\pi^{GC} = \frac{\lambda_{ss}{}^{2}\theta^{2/\gamma}(x_S - x_F)^2}{4} + x_F - nw \tag{A51}$$

Point *(i)* follows from (A20), (A41) and (A49). By comparing (A21) , (A42) and (A50) it is easy to see that $e_{ss}^{GC} > e_{ss}^{TC} = e_{ss}^{IC}$. This proofs point *(ii)*. Similarly, by comparing (A23), (A43) and (A51) it is easy to see that for $\lambda_{ss} > \lambda_i$ the following hold $\pi^{GC} > \pi^{TC} > \pi^{IC}$. This proofs point *(iii)*. □

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List of Tables and Figures

Figure 1- Principal (P), agents (A and B) and client (C).

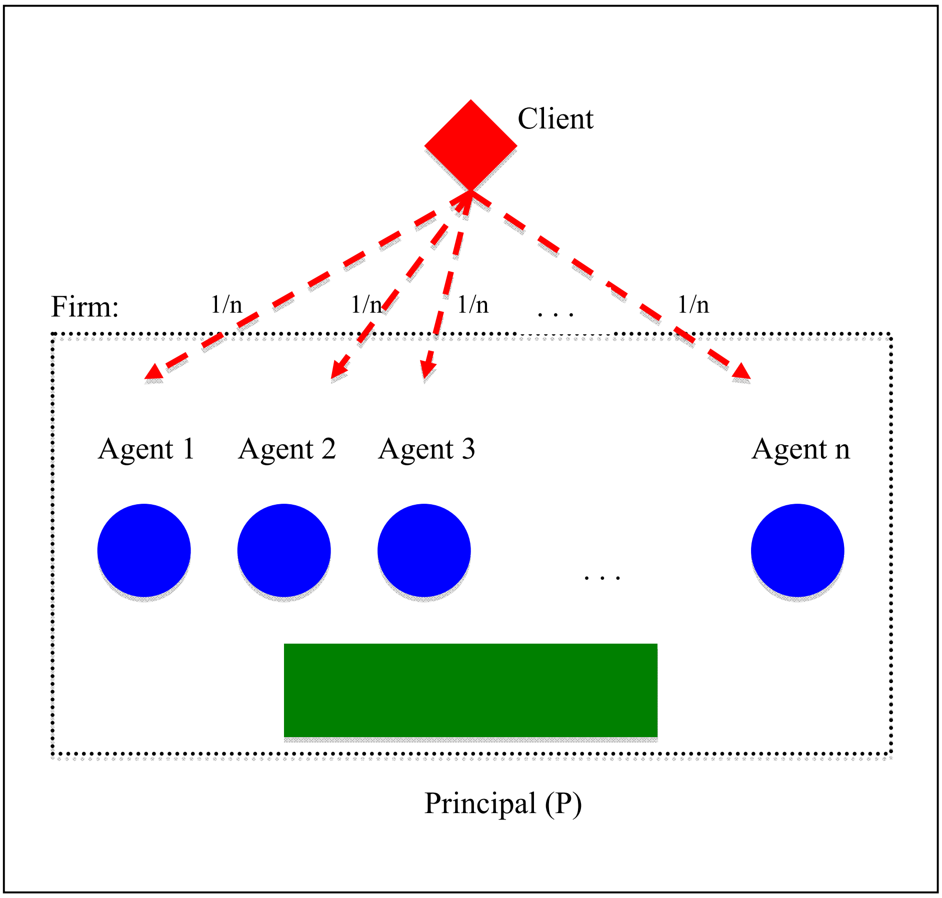

Table 1 – Players' Payments

Outcome	Agent's Payments IC [i,-i]	Agent's Payments TC [i,-i]	Principal's Payments
No one serves successful	$[w, w]$	$[w, w]$	x_F
j serves successful	$[w + b^{IC}, w]$	$[w + b^{TC}, w + b^{TC}]$	x_S

Figure 2 – Profit, effort and misallocation. The two curves represent the profit ($\underline{\pi}$) and effort ($\underline{e}$) under TC relative to the ones under IC for different degree of misallocation ($\mu(\lambda)$). For values greater than 1 the profit (effort) under TC is greater than the profit (effort) under IC. For the sake of simplicity the curves have been drawn by assuming $x_F = 2w$. The values associated with the three previous examples are reported with different colors. Notice: there exist a whole range of points for which TC is more profitable than IC, although it induces lower effort.